Instructions: Complete each of the following exercises for practice.

1. Compute the following iterated integrals.

(a) $\displaystyle\int_{x=1}^{4}\int_{y=0}^{2}(6x^2y-2x)\,dy\,dx$

(b) $\displaystyle\int_{y=0}^{1}\int_{x=0}^{1}(x+y)^2\,dx\,dy$

(c) $\displaystyle\int_{y=0}^{1}\int_{x=1}^{2}(x+e^{-y})\,dx\,dy$

(d) $\displaystyle\int_{x=0}^{\frac{\pi}{6}}\int_{y=0}^{\frac{\pi}{2}}(\sin(x)+\sin(y))\,dy\,dx$

(e) $\displaystyle\int_{y=-3}^{3}\int_{x=0}^{\frac{\pi}{2}}\big(y+y^2\cos(x)\big)\,dx\,dy$

(f) $\displaystyle\int_{x=1}^{3}\int_{y=1}^{5}\frac{\ln(y)}{xy}\,dy\,dx$

(g) $\displaystyle\int_{x=1}^{4}\int_{y=1}^{2}\left(\frac{x}{y}+\frac{y}{x}\right)dy\,dx$

(h) $\displaystyle\int_{y=0}^{1}\int_{x=0}^{2}ye^{x-y}\,dx\,dy$

(i) $\displaystyle\int_{t=0}^{3}\int_{\phi=0}^{\frac{\pi}{2}}t^2\sin^3(\sin(\phi))\,d\phi\,dt$

(j) $\displaystyle\int_{x=0}^{1}\int_{y=0}^{1}xy\sqrt{x^2+y^2}\,dy\,dx$

(k) $\displaystyle\int_{v=0}^{1}\int_{u=0}^{1}v(u+v^2)^4\,du\,dv$

(l) $\displaystyle\int_{t=0}^{1}\int_{s=0}^{1}\sqrt{s+t}\,ds\,dt$

2. Compute the double integral $\displaystyle\iint_R f(x,y)\,dA$ for function $f(x,y)$ and region R.

(a) $f(x,y)=x\sec^2(y)$; $R=[0,2]\times[0,\frac{\pi}{4}]$

(b) $f(x,y)=y+xy^{-2}$; $R=[0,2]\times[1,2]$

(c) $f(x,y)=\dfrac{xy^2}{x^2+1}$; $R=[0,1]\times[-3,3]$

(d) $f(x,y)=\dfrac{\tan(x)}{\sqrt{1-y^2}}$; $R=[0,\frac{\pi}{3}]\times[0,\frac{1}{2}]$

(e) $f(x,y)=x\sin(x+y)$; $R=[0,\frac{\pi}{6}]\times[0,\frac{\pi}{3}]$

(f) $f(x,y)=\dfrac{x}{1+xy}$; $R=[0,1]\times[0,1]$

(g) $f(x,y)=ye^{-xy}$; $R=[0,2]\times[0,3]$

(h) $f(x,y)=\dfrac{1}{1+x+y}$; $R=[1,3]\times[1,2]$

3. Compute the double integral $\displaystyle\iint_R f(x,y)\,dA$ for function $f(x,y)$ and region R.

(a) $f(x,y)=\dfrac{y}{x^2+1}$; $R=\big\{(x,y):0\le x\le 4, 0\le y\le\sqrt{x}\big\}$

(b) $f(x,y)=2x+y$; $R=\{(x,y):y-1\le x\le 1, 1\le y\le 2\}$

(c) $f(x,y)=e^{-y^2}$; $R=\{(x,y):0\le x\le y, 0\le y\le 3\}$

(d) $f(x,y)=y\sqrt{x^2-y^2}$; $R=\{(x,y):0\le x\le 2, 0\le y\le x\}$

(e) $f(x,y)=x\cos(y)$; R the region bounded by $y=0$, $y=x^2$, and $x=1$

(f) $f(x,y)=x^2+2y$; R the region bounded by $y=x$, $y=x^3$, and $x\ge 0$

(g) $f(x,y)=y^2$; R the triangle with vertices $(0,1)$, $(1,2)$, and $(4,1)$

(h) $f(x,y)=xy$; R the region bounded by $y=\sqrt{1-x^2}$ with $x,y\ge 0$

(i) $f(x,y)=2x-y$; R the radius 2 circular disk about the origin

(j) $f(x,y)=y$; R the triangle with vertices $(0,0)$, $(1,1)$, and $(4,0)$

4. Sketch the region of integration and then change the order of integration.

(a) $\displaystyle\int_{y=0}^{1}\int_{x=0}^{y}f(x,y)\,dx\,dy$

(b) $\displaystyle\int_{x=0}^{\frac{\pi}{2}}\int_{y=0}^{\cos(x)}f(x,y)\,dy\,dx$

(c) $\displaystyle\int_{x=1}^{2}\int_{y=0}^{\ln(x)}f(x,y)\,dy\,dx$

(d) $\displaystyle\int_{x=0}^{2}\int_{y=x^2}^{4}f(x,y)\,dy\,dx$

(e) $\displaystyle\int_{y=-2}^{2}\int_{x=0}^{\sqrt{4-y^2}}f(x,y)\,dx\,dy$

(f) $\displaystyle\int_{x=0}^{1}\int_{y=\arctan(x)}^{\frac{\pi}{4}}f(x,y)\,dy\,dx$

5. Evaluate the integral (**Hint**: Reverse the order of integration).

(a) $\displaystyle\int_{y=0}^{1}\int_{x=3y}^{3}e^{x^2}\,dx\,dy$

(b) $\displaystyle\int_{x=0}^{1}\int_{y=x^2}^{1}\sqrt{y}\sin(y)\,dy\,dx$

(c) $\displaystyle\int_{x=0}^{1}\int_{y=\sqrt{x}}^{1}\sqrt{y^3+1}\;dy\;dx$

(d) $\displaystyle\int_{y=0}^{2}\int_{x=\frac{1}{2}y}^{1} y\cos(x^3-1)\;dx\;dy$

(e) $\displaystyle\int_{y=0}^{1}\int_{x=\arcsin(y)}^{\frac{\pi}{2}}\cos(x)\sqrt{1+\cos^2(x)}\;dx\;dy$

(f) $\displaystyle\int_{y=0}^{8}\int_{x=\sqrt[3]{y}}^{2} e^{x^4}\;dx\;dy$